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Dispersion in the enstrophy cascade of two-dimensional decaying grid turbulence

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The use of vertically flowing soap films allows one to obtain decaying two-dimensional turbulence with an enstrophy cascade range. Here we use this property to study the dispersion of a fine column of slightly heated liquid entering the turbulent flow. The width of this column increases with time in an exponential manner consistent with theoretical predictions for the average dispersion of particle pairs despite the multiparticle nature of the dispersion studied. Other features such as Gaussian mean profiles of the temperature across the channel width are also evidenced.

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A smoke puff or cloud coming out of a chimney and entering a turbulent flow as on a windy day will spread violently under the action of the external turbulent flow. It has long been recognized that the average spreading of a smoke puff or cloud in a fully developed turbulent flow gives direct information on the transport properties of turbulence.The understanding of dispersion and therefore the transport properties of turbulent flows is of central importance in nonlinear physics and hydrodynamics and is of utmost importance in understanding the dispersion of pollutants in the air or nutrients in the oceans. Richardson [1], Taylor [2], and later Batchelor [3], who discussed both single particle diffusion and relative diffusion, proposed that the spreading process is governed by the properties of the turbulence itself and gives direct information on the correlations of the velocity field. Later, Gifford [4] used similar ideas to analyze the spreading of smoke puffs, a large collection of particles, in turbulent air and found evidence for Richardson dispersion, applicable for pairs of particles, for the average spreading of the smoke puff. Such measurements are of the Lagrangian type as opposed to the usual Eulerian measurements in turbulence where the velocity is measured in a single location. Lagrangian measurements tracking single particles or pairs of particles are rare and have appeared only recently in two dimensions [5] (in the inverse cascade range) and three dimensions [6,7]; these latter measurements provide not only a test of the Kolmogorov theory of turbulence but also point out causes for departures from it. The theory of Lagrangian dynamics of turbulence has however made great progress for some particular situations where explicit calculations have been carried out [8]. Our measurements are relevant to multiparticle dispersion where predictions and experiments are very rare.

The experiment reported here is a two-dimensional realization of the spreading of a "cloud." The use of soap films falling between two parallel wires under the action of gravity allows to obtain a decaying turbulent field behind an array of cylinders as has been demonstrated in several experiments [9]. The turbulence found has velocity statistics in close agreement with theoretical predictions of the enstrophy cascade [10]. Therefore, our experiment allows to study turbulent dispersion in the enstrophy cascade range of two-dimensional (2D) decaying turbulence. The intensity of the turbulence behind the grid can be controlled by changing the flux of soap water feeding the film. As the flux increases the mean velocity and the turbulent intensity increase. A fine laminar column of slightly heated soap water, in such a way as to be "visible" to an infrared camera, is injected into the soap film at the level of the grid producing the turbulence as shown in Fig. 1. The injection flux for the column was kept much smaller than the flux feeding the soap film to introduce as little changes to the basic flow as possible. Figure 1 also shows the effect of column injection flux: for a relatively high column flux, the column which is thicker than the film does not mix well and remains thick but fluctuates violently, while for a smaller column flux (the regime we examined), the column mixes relatively quickly with the surrounding film. The central observation of this experiment is summarized in Fig. 1. At low film flux (low turbulence intensities) the column stays intact and spreads very little. For higher film flux and therefore higher turbulence intensities, the column spreads significantly and seems to reach the sides of the channel sufficiently far downstream. These images are obtained with an infrared camera and several photographs were averaged to obtain the time averaged temperature field in the film. The intensity of the images obtained with the infrared camera gives the temperature field in the film directly. The temperature is highest at the injection point and decreases as the column proceeds downstream and spreads. The temperature profile at a fixed vertical distance from the grid is well described with a Gaussian function centered at the center of the channel. Our central result is that the width of these Gaussian temperature profiles varies roughly exponentially with the distance from the grid or equivalently with time after the passage of the grid since the mean flow present in this experiment advects the structures downstream. This exponential law is the 2D analog of the Richardson law for pair dispersion in three dimensions which can be used as suggested by Batchelor and later by Gifford to the dispersion of a whole cloud or large ensemble of particles as is the case here, even though the applicability of this law for a large number of particles remains without a proof. This is to our knowledge one of the first experiments of dispersion in the enstrophy cascade. Note that the exponential dispersion law was verified for pairs of particles for large scale atmospheric turbulence [11] pointing to its importance in natural flows.

The soap films used are drawn on a frame of nylon wires of 2 m in length and 5 cm in width. The flux of water feeding the film was controlled with a micropump which delivers

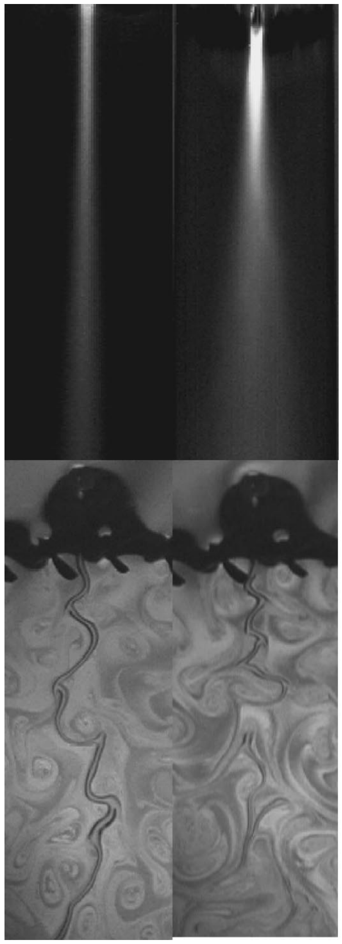

FIG. 1. Photographs using an infrared camera of the heated column in low flux and low turbulent intensity (upper left: film flux of 0.15 ml/s, column flux: 0.08 ml/s, injection temperature: 23.4 °C) and high flux and high turbulent intensity (upper right: film flux of 0.6 ml/s, column flux: 0.08 ml/s, injection temperature: 29.9 °C). The width of the photographs is the channel width of 5 cm. The two bottom photographs are taken in reflection of white light for a flux of 0.4 ml/s but two different column flux of 0.12 ml/s (left) and 0.09 ml/s (right). Note that the column "dissolves" in the surrounding film a few centimeters from the grid for the small flux but the column remains thick but fluctuating for the higher flux.

from 0.1 to 1 ml/s. The speeds obtained varied between 1 and 3 m/s and the film thickness between 1 and 5 μm. When a grid (horizontal array of seven cylinders of 3 mm diameter spaced by 6.5 mm) is inserted perpendicularly to the film, the mean speed decreases slightly and the turbulence intensity could be varied from about 2% to 15% by varying the film flux. The column of slightly heated soap water (between 3 and 12 °C above the temperature of the film solution) was injected using a small diameter (1 mm) pipette at the level of the grid. The flux of this column was controlled with a second micropump connected to a second soap reservoir kept at the desired temperature with a small heater. Basically the flux of the column was ten to five times smaller than the main flux of the soap film so as to have a minimal disturbance of the basic flow. As we will show below, we checked that differences in temperature and differences in flux did not change the results. One may worry that large temperature gradients may generate large surface tension gradients, for example, or that the flow of the column introduces its own dynamics into the flow field. By choosing a small injection flux and relatively small temperature differences we minimized these effects and checked that the flow was not perturbed by the injection of the column as we will see below. The velocity was monitored with an LDV system (Dantec) capable of giving time traces of the longitudinal and transverse components of the velocity with a temporal resolution better than 1 ms. The seed particles were polystyrene spheres of 0.8 μm. Measurements of the velocity fluctuations allow us to measure the statistics of the velocity field and determine the nature of the flow directly in the experimental configuration used to study the dispersion of the heated column. The dispersion of the heated column was monitored with a sensitive infrared camera (Thermacam PM 595 with a resolution of 0.1 °C at 30 °C and a spectral response between 7.5 and 13 μm) which gives the temperature field in the film directly. For each configuration several images were used to obtain the temporally averaged temperature field (the acquisition time of the camera is long and is around 0.05 s so some averaging already occurs in a single image). This temporal averaging is crucial to the study of the average dispersion of the column. In order to suppress possible effects due to the background, we have subtracted the image without a column present. As shown in Fig. 1, when the turbulence intensity is low (obtained for a low film flux of 0.15 ml/ s with a speed of 1 m/s and a turbulence intensity of less than 3%) the column shows basically no sign of significant spreading. The second image taken at a film flux of 0.6 ml/s with a mean speed of about 2 m/s and higher turbulent intensity (about 10% at a location of 10 cm from the grid) shows a much more significant spreading of the column in the transverse direction of the flow. For this situation, the column itself seems to dissolve in the turbulent flow as seen in Fig. 1. Clearly the turbulence is the major effect causing the spreading of the fine column. Along with this spreading the temperature at the center of the column decreases with downstream distance from the grid. Here we will draw an analogy between downstream distance from the grid (denoted Y) and time of evolution t. Basically the structures are generated at the grid position and are convected downstream by the mean velocity V; t is then given by Y/V. This is an explicit use of the Taylor frozen turbulence assumption which was found to work remarkably well in these turbulent soap films [12]. The velocity spectra of the produced turbulence are shown in Fig. 2 for different positions from the grid. Clearly a -3.3 scaling exponent is observed for both components of the velocity at different positions from the grid for a range of scales between about 4 mm and 2 cm. A slight decrease of the amplitude is seen with distance from the grid indicating that the turbulence is decaying. Since the spreading of the column is seen for distances from the grid going up to 15 or 17 cm, this decrease should be kept in mind since it indicates that the rate of enstrophy transfer β decreases with distance from the grid. In two dimensions as is the case here the energy density spectrum is predicted to scale as k^{-3} (modulo logarithmic corrections, k is the wave

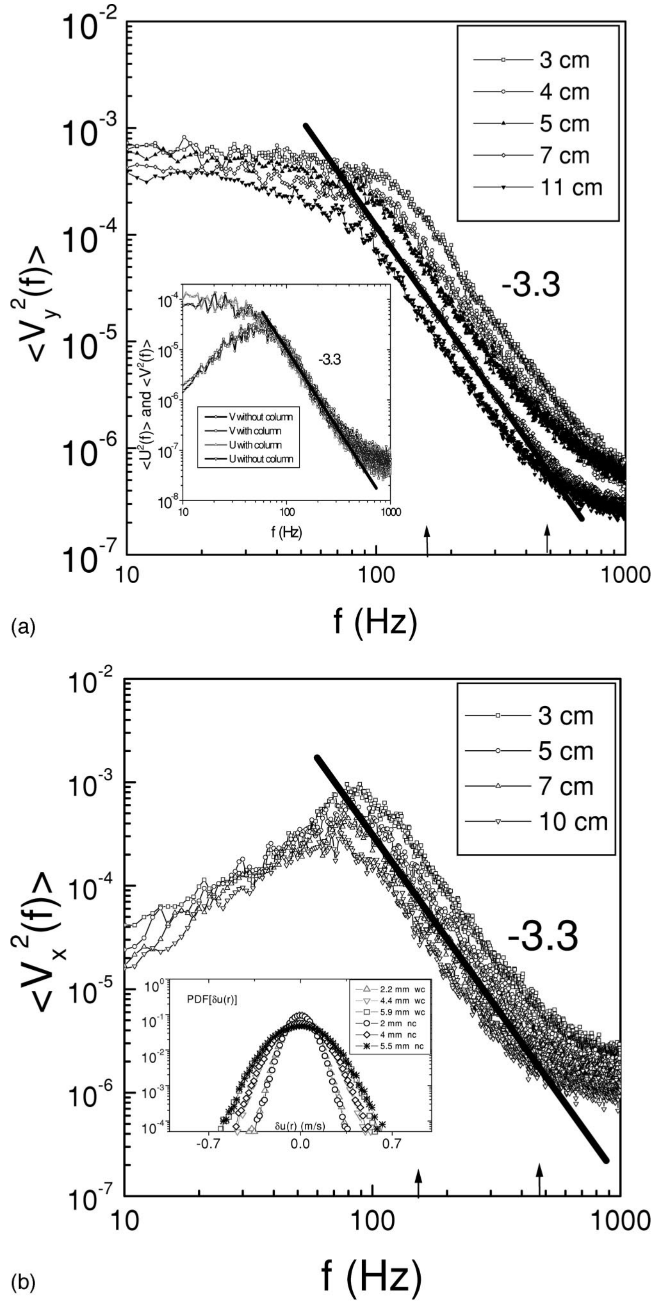

FIG. 2. Longitudinal (a) and transverse (b) velocity power spectra (at different positions from the grid for a flux of 0.4 ml/s). The two arrows indicate the position of the shedding frequency and the cylinder diameter which are at 160 Hz and 500 Hz, respectively. Insets: Comparison between velocity spectra taken with and without the column present. Probability density functions of longitudinal velocity differences across different increments r with and without the column.

number) in the so-called enstrophy cascade range, i.e., for scales smaller than the injection scale of the turbulence. This is basically the case here. The inverse cascade, which occurs for scales larger than the injection scale, and for which the energy density spectrum scales as $k^{-5/3}$ is not observed here. Supposedly, this scaling is absent for decaying turbulence [13]. The two arrows in the graph indicate the scales associated with the diameter of the cylinders in the grid used and the position of the shedding frequency from the individual cylinders. The injection scale, taken as the crossover point between the scaling regime and the large scale anisotropic regime seems to correlate with the shedding frequency of the cylinders rather than with the cylinder diameter. The injection of the column does not modify the flow: the velocity spectra with and without the column present are basically similar as seen in the inset of Fig. 2. Also, the probability distributions of velocity increments remained similar with and without the column present (inset Fig. 2) indicating that the column did not influence the velocity field markedly, at least within the precision of the measurements.

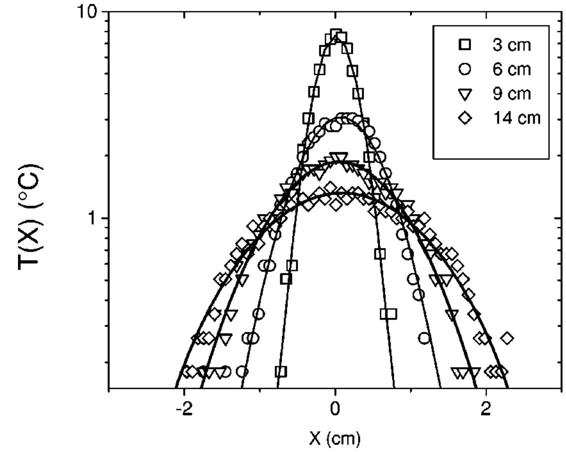

FIG. 3. Temperature profiles across the channel for different vertical positions Y. The flux is 0.6 ml/s, the column flux is 0.08 ml/s, and the injection temperature is 29.9 °C.

A description of the properties of the column dispersion is summarized in Fig. 3 which shows temperature profiles along the transverse direction X taken at different positions from the grid for a high turbulence intensity. Note that as the downstream distance Y increases the maximum temperature at the center of the channel decreases while the width of the profile increases. The area under these profiles however changes little. A maximum of 20% reduction is observed between about 1 cm and 20 cm from the grid. Since the most important decrease of this area occurs very near the grid it corresponds to the important decrease of the turbulent intensity observed at distances up to 1 or 2 cm from the grid. Thus, leakage to the surrounding air is minor. It should be noted that at distances up to 2 cm the flow still presents the signature of shedding from individual cylinders (the shedding frequency is indicated by an arrow in Fig. 2). The column itself keeps its integrity down to 1 or 2 cm from the grid before being well mixed with the rest of the flow. It should be noted that this mixing occurs more rapidly for columns with smaller flux and for high turbulence intensities which is the case for the presented results.

The average temperature profiles along the X direction are well described by Gaussian functions as can be seen in Fig. 3 where Gaussian fits are superposed upon the experimental temperature profiles. This functional shape has usually been used for the variation of the density of smoke puffs without much justification nor independent checks [4]. The maxi-

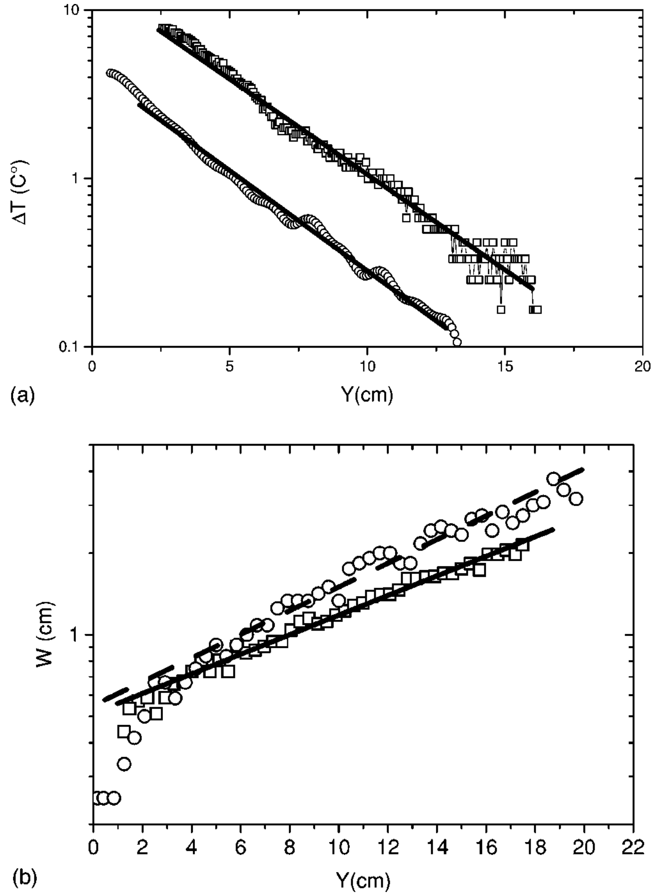

FIG. 4. (a) Decrease of the maximum temperature at the center of the channel vs downstream distance *Y*. Two runs are presented: upper curve: flux of 0.6 ml/s, column flux: 0.08 ml/s, injection temperature: 29.9 °C, lower curve: flux of 0.45 ml/s, column flux: 0.08 ml/s, and injection temperature of 24.7°C. (b) Increase of the width of the temperature profiles vs downstream distance *Y*. Circles: flux: 0.5 ml/s, column flux: 0.08 ml/s, injection temperature 34 °C. Squares: flux 0.55 ml/s, column flux 0.07 ml/s, and injection temperature: 24.6 °C.

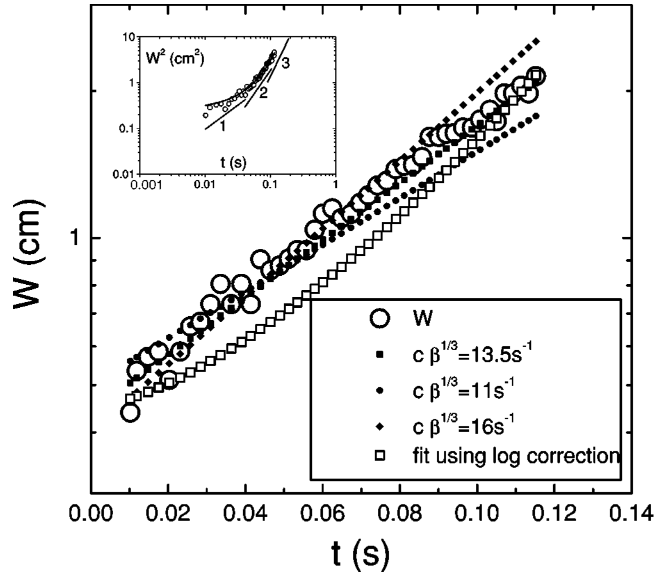

FIG. 5. Increase of the width of the temperature profiles vs time ($t=Y/V$ where V is the mean velocity) along with different fits using the expression in the text (flux 0.55 ml/s, column flux 0.07 ml/s, and injection temperature: 24.6 °C). Inset: a test of a power law increase of W vs t.

mum temperature at the center of the column (corresponding to the center of the channel as we injected the hot water from a pipette located at the center of the channel) decreases roughly exponentially as the downstream distance Y increases. Similarly, and this is our central result, the width of the Gaussian temperature profiles, W, increases roughly exponentially versus downstream distance Y. Both these results are shown in Figs. 4(a) and 4(b). This functional form was observed for different configurations or more precisely for different combinations of column temperature, column flux, and film flux. The exponential form for the increase of the temporally averaged width of the temperature profiles is robust and was reproduced under different conditions. The downstream distance can be converted to a time of evolution t using $t=Y/V$ as indicated above. The mean speed at different locations was measured using LDV; the speed changes from 1.35 m/s to 1.6 m/s for distances between 2 and 20 cm from the grid (the velocity profile across the channel is basically flat with a decrease at distances of less than 5 mm from the sides). This conversion is done in Fig. 5 showing the exponential increase of the width versus time. As has been argued by Batchelor and Gifford for smoke puffs, one can analyze the dispersion of a large number of particles using notions from particle pair dispersion. These arguments suggest that the temporal evolution of the averaged width of the smoke puffs is similar to that of the separation of particle pairs. This would suggest that the exponential increase of the average width is consistent with the dispersion law for particle pairs in the enstrophy cascade of 2D turbulence, calculated by Lin in 1977 [14]. His argument, which uses the same reasoning as for pair dispersion in three-dimensional turbulence and which gives the Richardson dispersion law, gives the following law for the average dispersion rate of particle pairs: $\langle R^2(t)\rangle \propto R_0^2 \exp(2c\beta^{1/3}t)$ where R_0 is the initial separation, c is a numerical constant, and β is the enstrophy injection rate. (Note that an exponential growth is expected not only in the enstrophy cascade but for any smooth velocity field such that the velocity increments scale linearly in the distance r). Fits using this law to our experimental data for W versus t give values of $c\beta^{1/3}$ between 10 and 20 s^{-1}. *A priori* the constant c is not known. However from measurements of the correlation function $\langle \delta u(r)\delta\omega^2(r)\rangle$ (using two LDV probes; see Ref. [9] and references therein for details) for which exact calculations give a result of $-2\beta r$, we estimated a value of $\beta^{1/3}$ of 40 $s^{-1}$for the same conditions as for Fig. 5. This estimate of the rate of enstrophy transfer is in agreement with previous determinations [9] and suggests a value of 1/3 for the constant c. As mentioned above, the turbulence intensity and therefore β vary with distance from the grid. However, the estimated decrease of $\beta^{1/3}$ from our spectral measurements indicates variations of no more than a factor of 1.5. For this matter, we have superposed in Fig. 5 fits with $c\beta^{1/3}$ values slightly higher and slightly lower than the average value giving an error of about

$\pm 2.5s-1$. The variation of β with distance falls roughly within this error margin. This figure also shows a fit using a slightly modified form for the increase of the width. This functional form is obtained for the case where logarithmic corrections are considered for the energy density spectrum in the enstrophy cascade. This form reads: $\langle R^2(t)\rangle \propto R_0^2 \exp(Kt^{3/2})$. As can be seen in Fig. 5, the simple exponential fit works much better than this latter form. Apparently, logarithmic corrections are not seen in the variation of the width versus time. In the inset of Fig. 5, we have also tested the possibility that the increase of the width (or rather the square of the width) is a power law function. As can be seen here, the power law increase is not justified. Recently, Goldburg *et al.* [15] studied the dispersion and widening of a von Karman street in a turbulent soap film just like in our experiments. Their data suggested a diffusive behavior for the increase of the width of the von Karman street. This would correspond to an exponent of 1 in our inset which probably hugs the first points at early time, but the deviation is considerable at larger times. We believe that our data is more extensive so we could discern differences between a power law increase and an exponential increase. In any case, the two experiments do not measure similar quantities. Note that numerical simulations [16] of relative dispersion in two dimensions have found only a rough agreement with this law for a small time interval and for very small initial separations compared to the injection scale. In our experiments, the initial separation is at least ten times smaller than the injection scale. Earlier simulations also displayed a small time interval where this law seems to be valid [17] with possible corrections due to the logarithmic correction mentioned above.

A surprising aspect of our experiments is that the spreading of a large collection of particles, as represented by the Gaussian temperature profile, follows similar laws as that of pairs of particles. Typically, to test Richardson like dispersion, one releases pairs of particles with a fixed initial separation and one measures the particle pair separation versus time. This temporal variation of the pair separation is then averaged over many realizations. The final averaged pair separation versus time is what follows the Richardson law. Here, a collection of particles is injected as a column with small radius. In a sense, our cloud here is a horizontal segment of this column, and its average spreading is measured and averaged over time. Our particle pair here is represented by the width of the Gaussian temperature profile. The reasons why this width behaves as if it were an average particle pair separation are not clear to us and only further theoretical work on multiparticle dispersion could give an answer to this question. Nonetheless, our results point to some simplifying aspects of this difficult problem and suggest that the intuition of Batchelor and Gifford may well have experimental support.

Another important point to keep in mind concerning this experiment is the importance of intense events in turbulent dispersion. By averaging, the large fluctuations of the temperature profiles get smeared out and an averaged profile results. By examining the temperature profiles from individual images (i.e., before averaging of several images), large excursions from the average profiles are clearly visible and are due to intense events in the turbulence studied which is capable of dispersing the heated zones much farther than the measured averaged dispersion. These images are not instantaneous as the acquisition time of the camera is 50 ms, so some averaging has already occurred. These large fluctuations are therefore present and are even more so were we to use a faster camera so that little averaging occurs over the time the image is taken. Unfortunately this is not possible with the infrared camera we used. It may well be that such intense events modify the tails of the temperature profiles which may depart from the Gaussian shape. Much better temperature resolution is needed to evidence such effects. However this is not the main point here since the main concern is the average dispersion of the column and not the fluctuations of the temperature field. An important point here is that averaging of several pictures does converge to a single well defined temperature profile across the channel. In addition, the evolution of this width versus time shows a well defined behavior which can be explained by simply thinking of the average width as an average distance between two imaginary particles.

By monitoring the average dispersion of a slightly heated laminar column introduced in a quasi-2D turbulent flow obtained using a soap film, we have evidenced an exponential dispersion law for the width of the column. This exponential variation is in agreement with the predicted dispersion law for turbulence in the enstrophy cascade range. As suggested long ago, such dispersion laws valid for pairs of particles may be generalized to the average dispersion of large collections of particles such as smoke puffs or columns which is the case here. Our experiments bring hope that multiparticle dispersion which is an open issue in turbulence, may present some simplifying aspects.

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